1836MNRAS...4....5L

are to be attributed to errors in the catalogues of Bradley or Piazzi (for Mr. Baily has examined the computations by which our catalogue is *deduced* from theirs), or whether there is some irregular motion in the stars themselves, time will shew. It is from such undertakings as this of Mr. Wrottesley, and so executed, that we must expect to fix the state of the heavens at certain epochs, and so prepare the data for future speculation and future discovery. For such inquiries, Mr. Wrottesley's present of the original transit books, to which the partial mean places serves as a complete index, will be of great and permanent value.

There are several remarks which deserve, and doubtless will receive, the attention of practical astronomers, but which would be here out of place. We must not, however, omit mentioning that the whole work has been performed, to use Mr. Wrottesley's words, "without any foreign aid," by himself, and, under his superintendence, by his assistant, Mr. John Hartnup; upon whom, indeed, in consequence of Mr. Wrottesley's frequent and continued absence from home, the task of observing and computing chiefly fell, and who has executed this task with extraordinary zeal, skill, and fidelity.

III. On the Projection of Maps and Charts. By Professor Littrow.

Three kinds of projections are chiefly had recourse to in the construction of maps,—the *orthographic*, the *stereographic*, and the *central*. Each has its peculiar advantages and disadvantages in particular cases; but as the stereographic possesses two very remarkable properties,—1st, that all circles of the sphere are projected either into circles or straight lines; and, 2nd, that the angles made by two projected circles, are equal to the angles made by those circles on the sphere,—it has been most frequently employed in the construction of planispheres.

All projections of the sphere on a plane have this disadvantage, that they distort the map in a greater or less degree. With a view to lessen the defect, various modifications of the stereographic projection have been proposed, such as that of La Hire, who supposed the eye to be situated out of the sphere, and at a distance from its convex surface equal to the sine of 45°. But, though by this means the distances are less altered on the map, the great advantages of the stereographic projection are lost. The circles of the sphere become in general ellipses or hyperbolas, and the projections of the meridians and parallels no longer intersect at right angles.

Though it is convenient to employ the principles of perspective in representing spherical surfaces on a plane, there is nothing in the nature of the thing itself which renders such a method of proceeding necessary. Let the meridians and parallels be represented on the map by any lines whatever, the representation will still be accurate if the different points are laid down so as to have the same relations to those lines as the points represented have to

1836MNRAS...4....5L

the corresponding circles of latitude and longitude. The construction of a map in this way is evidently an indeterminate problem; but it is rendered determinate by subjecting the representation to certain conditions which may have no reference to perspective. Mercator's chart is an example. The condition to be fulfilled in this chart is, that the rhumbs shall be straight lines, and make with each other the same angles as on the sphere. This condition determines the well-known property of the chart, namely, that the degrees of longitude are all equal, while the degrees of latitude are inversely as the sines of the polar distances.

It is stated by Lagrange, that Lambert was the first who considered the theory of the construction of maps under this general view. The problem which Lambert undertook, was to determine the lines representing the meridians and parallels, so that all the angles on the map should be equal to the corresponding angles on the sphere. This problem is extremely important, because it follows, from the equality of the angles, that any infinitely small portion of the surface of the sphere, and its representation on the map, are similar figures, and, consequently, that small parts represented differ from the original only in respect of magnitude. Lambert's solution is given in Vol. III. of his *Beytrage*, &c. The same problem was undertaken by Euler in the *Petersburg Memoirs* for 1777; but neither of these illustrious geometers prosecuted the subject further than to shew, that the known properties of the stereographic projection, and Mercator's chart, were comprehended in their general solution.

The *Berlin Transactions* for 1779 contain two memoirs, by Lagrange, on the same subject. In the first, he gave a new solution of the problem of Lambert, of which he shewed the stereographic projection to be only a particular case; and, in the second, undertook the more difficult question of determining the form of the arbitrary functions which enter into the general solution, so that the lines representing the meridians and parallels shall be of a given nature.

Gauss also, in answer to a prize question proposed by the Royal Society of Sciences of Stockholm, has given a memoir on this subject, which was published in No. 3 of *Schumacher's Abhandlungen*, and a translation of it in the *Philosophical Magazine* for August and September, 1828. Instead of confining himself, however, to the particular case of the representation of a spherical surface on a plane, he undertook the solution of the general problem, namely, to represent the parts of any given surface on any other given surface, so that the differential elements of the first surface shall be similar to their representations on the second. Having determined, by a very beautiful analysis, the differential expressions from which the co-ordinates of any point of the map are found, in terms of the co-ordinates of the corresponding point of the primitive surface, he applies his general solution to the particular cases in which it is required to represent a plane surface on a plane, the surface of a cone on a plane, of a sphere and

1836MNRAS...4....7L

ellipsoid on a plane, and, lastly, of an ellipsoid of revolution on the surface of a sphere.

The object of Professor Littrow is to deduce the general properties of the three principal projections, which, though they differ from each other in no other respect than in the situation of the eye and perspective plane, with regard to the principal circles of the sphere, have hitherto been always treated as distinct and independent problems. The manner in which he accomplishes this may be easily described.

He supposes the eye to be situated at any distance above the sphere, and the plane of projection to be inclined at any angle to the straight line drawn from the eye to the centre. Having assumed x and y as the rectangular co-ordinates of any point on the map, he chooses for the axis of x the intersection of the plane passing through the eye, the centre of the sphere, and the pole with the plane of projection, and finds expressions for x and y in terms of the latitude and longitude of the corresponding point on the sphere. Two equations are thus obtained. Eliminating from these two equations the latitude, the resulting equation is that of the projection of the meridian passing through the projected point; and, by eliminating the longitude, he gets the equation of the parallel circles. He then applies this general solution to each of the three projections successively, finds in each case the equations of the projections of the meridians and parallels, and shews the cases in which they are circles, or straight lines, or any of the three conic sections. He then considers the projection of the spheroid of revolution, and gives the formulæ which express the latitude of any point on the sphere in terms of the latitude of the corresponding point on the spheroid.

The concluding part of the memoir contains some general remarks on the solutions of Gauss and Lagrange; and a demonstration that Gauss's formulæ are comprehended in those of Lagrange, the latter being only particular values of the former. The analysis is extremely neat, symmetrical, and perspicuous; and the subject, as before remarked, one of much interest.

IV. On the construction of the Hour-lines of Sun Dials. By Professor Littrow.

The remarks which Professor Littrow makes on this subject, and his method of treating it, are precisely analogous to those employed in the preceding paper. Indeed, the two subjects are closely related, as the whole theory of Dialling may be deduced from the properties of the central projection of the sphere. He begins by remarking that, though the subject has been very frequently treated of by others, the problem has not hitherto been resolved with all that generality of which it is susceptible, and which it deserves in so eminent a degree; that what regards the length of the shadows has never been satisfactorily handled; and that with regard to dials on curve surfaces, nothing complete, in point of theory, has